Sheet # 1
Introduction to Number Theory

1. Show that 3|99, 5|145 and 888|0.

2. Determine which of the following integers are primes:
   (101 – 107 – 113 – 103 – 111 – 121)

3. Find the greatest common divisor of each of the following pairs of integers:
   a) 15, 35
   b) 0, 111
   c) 99, 100
   d) -12, 18

4. Find the prime factorization of: 36 – 222 – 5040 – 39 –256.

5. Find the *gcd* of the following pairs of integers:
   a) $(2^2 . 3^3 . 5^5 . 7^7), (2^7 . 3^5 . 5^3 . 7^2)$.
   b) (2. 3. 5. 7. 11. 13), (17. 19. 23. 29).

6. Find the least non-negative residue modulo 13 of:
   (22 -1 100 -100 -1000 1001).

7. Find the least positive residue of :
   a) 3^{10} mod 11
   b) 5^{16} mod 17
   c) 2^{12} mod 13
   d) 3^{22} mod 23

8. Using Fermat's theorem, find 3^{201} mod 11.

9. Find a reduced residue system modulo: 6, 14, 9, and 17.

10. Using Euler's theorem, find the least positive residue of 3^{100000} mod 35.

11. Using Euler's theorem, find:
    a) The last digit in the decimal expansion of 7^{1000}.
    b) The last digit in the hexadecimal expansion of $5^{1000000}$.

12. Solve the following linear congruences using Euler's theorem:
    a) $5x \equiv 3 \pmod{14}$.
    b) $4x \equiv 7 \pmod{15}$.

13. Using Euclid's algorithm to find:

a) gcd (24140, 16762).

b) gcd (4655, 12075).

14. Determine the order of:

a) 2 mod 5.

b) 3 mod 10

c) 10 mod 13.

15. Find a primitive root (a generator) modulo: 4, 5, and 14.

16. Show that the integer 12 has no primitive roots.

17. How many incongruent primitive roots are for 13?

Best Wishes of Success